

SECONDARY ISOTOPE EFFECTS AS PROBES FOR FORCE CONSTANT CHANGES

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INTRODUCTION

An isotope effect on a rate is referred to as a secondary isotope effect if the locus of the isotopic substitution is not involved in the bond breaking or the bond forming of the reaction. While this discussion will be largely devoted to such isotope effects on rates, one can define and treat secondary isotope effects on equilibria in a similar manner.

There has been considerable discourse in the literature about the possible causes of secondary isotope effects. We mention, without discussion, the following: hyperconjugation¹, inductive effects² and non-bonded interactions³. These arguments can, in general, be translated into a description of a difference in force fields between reactants and transition states and thus into a formulation of isotope effects within the framework of transition state theory which has been previously described⁴. The present discussion will undertake an examination of secondary isotope effects from the point of view of the force fields of the reactants and the transition states. We will not consider the details of how the changes of force field between reactant and transition state come about, nor will we try to rationalize these changes. The computations were carried out within the harmonic approximation although some of the above-mentioned mechanisms for secondary isotope effects do certainly involve changes in anharmonicity contributions.

Calculations will be presented for isotope effects in some model systems. In the model reactions the reactant is RX where R is an alkyl group and X possesses a mass of eighty. Bond distances were chosen as follows: $CH = 1.09 \text{ \AA}$, $CC = 1.54 \text{ \AA}$, and $CX = 1.91 \text{ \AA}$ with all angles tetrahedral. Two types of transition states have been considered: (i) RX^\ddagger with the RX bond broken, sometimes designated by $(R-X)^\ddagger$, or with a CC bond broken. The geometry of RX^\ddagger was usually the same as the corresponding reactant although in some calculations the broken bond was elongated (with little influence on the isotope effect). Some other geometry changes are explicitly discussed. A zero frequency of decomposition ν_L^\ddagger was generally obtained by setting the relevant stretching force constant equal to zero with no interaction force constants between this stretching coordinate and other coordinates. Other ways of obtaining a ν_L^\ddagger and other values of ν_L^\ddagger were not investigated in any detail. (ii) RX_2^\ddagger with the geometry about the carbon atom

† Research at Brookhaven National Laboratory carried out under the auspices of the United States Atomic Energy Commission.

to which the two X atoms are attached, a trigonal bipyramid, unless otherwise mentioned†. The CX distances were usually chosen 2.25 Å. A zero frequency ν_{L^\ddagger} was obtained either by setting $f_{CX_1^\ddagger} = f_{CX_2^\ddagger} = f_{12^\ddagger}$, where f_{12^\ddagger} is the interaction force constant between the two CX motions, or by setting just one of the CX stretching force constants equal to zero as in (i) above. Other ways of obtaining ν_{L^\ddagger} and other values of ν_{L^\ddagger} were again not investigated in detail. We can, however, state that in many cases we have obtained imaginary frequencies with both of the above types of transition states by either setting $f_{12^\ddagger} > f_{CX^\ddagger}$ or by setting the previously zero force constant negative, and that the results so obtained lead to no differences in our general conclusions. The two reaction types bear some similarity to S_N1 and S_N2 reactions, respectively. X might be a bromine atom but no particular significance should be attached to its mass except for the fact that it is much heavier than an H atom. The fact that the bending force constants coupling X to the alkyl group were chosen of "reasonable magnitudes" enables one to think of X, at least in a qualitative manner, as any heavy group, even an alkyl group. The discussion on cut-off and localization of isotope effect in the previous paper⁴ also enables one, again at least qualitatively, to carry the arguments concerning the present models over to other systems. As in our previous calculations⁴, reasonable force constants were used for the reactants. No attempt was made to find "best" force fields. In some cases, details of force fields have been described below the Tables. It will be seen that somewhat different force fields were used on different occasions. Reference should be made to the previous discussion⁴ for the philosophy which underlies the model calculations. We want to emphasize again that, in general, we are not attempting to reproduce any experimental results nor are we trying to predict results for any specific system. We *are* attempting to explore the nature of secondary isotope effects from a fairly general point of view.

The numerical calculations were carried out with the use of a fast digital computer in the manner which has been previously described⁴. Tunnelling is neglected. Tunnelling should actually not be of great importance in secondary isotope effects even when they involve hydrogen *versus* deuterium. Secondary effects are expected to be, and are found to be, considerably smaller than primary ones. For the most part, only secondary hydrogen isotope effects have been experimentally studied, and these will be the ones principally discussed here.

A SIMPLE EQUATION

In the previous paper⁴, a very simple equation for the isotope effect has been presented:

$$\ln(k_1/k_2) = \ln(\nu_{1L^\ddagger}/\nu_{2L^\ddagger}) + (\bar{\nu}/24) (\hbar/kT)^2 \left\{ \sum_{i,j} f_{ij}(g_{1,ij} - g_{2,ij}) - \sum_{i,j} f_{ij}^\ddagger(g_{1,ij} - g_{2,ij})^\ddagger \right\} \quad (1)$$

where the first double sum is over the internal coordinates of the isotopically substituted reactant while the second one is correspondingly over those of

† The geometry around this carbon atom is usually: XCX angle = 180°, H(C)CX angles = 90°, H(C)CH angles = 120°, R(CH) = 1.09Å, R(CC) = 1.54Å. In other ways the geometry of R is the same as in reactant.

the transition state (\ddagger). 1 and 2 refer to light and heavy isotopic species, respectively; $\nu_{1L}^\ddagger/\nu_{2L}^\ddagger$ is the isotopic ratio of the "frequencies of decomposition" (it is also the high temperature limit of the isotope effect and the classical limit⁴); f_{ij} refers to the force constant involving the internal coordinates q_1 and q_j (*i.e.*, $2V = \sum f_{ij}q_1q_j$); g_{ij} refers to the corresponding element of the Wilson G-matrix⁵ which can be evaluated on the basis of a knowledge of molecular geometry and atomic masses. Equation (1) with $\bar{\gamma} = 1$ is valid for all isotope effects at sufficiently high temperatures. It was sought to extend its validity to lower temperatures by the introduction of a temperature-dependent $\bar{\gamma}$ (≤ 1)⁶. We have shown that there may be difficulty in finding the magnitude of $\bar{\gamma}$ and that there are cases in which the use of equation (1) may not be meaningful at all⁴. Yet we find equation (1), which expresses the isotope effect directly in terms of the force constants of reactant and transition state, an extremely useful starting point in our thoughts about secondary isotope effects.

In using equation (1), only those terms in the summations need be included in which the coordinates q_1 and q_j both involve a particular isotopic atom, for only then is $g_{1, ij} - g_{2, ij} \neq 0$. While the following argument may be presented under somewhat more general conditions, we will now assume that the part of the force field involving the isotopic positions is adequately expressible in terms of just diagonal force constants; *i.e.*, we use a valence force field. The only coordinates needed to describe the systems with which we are dealing are bond stretches, bond-angle bends, and torsions. The force constants for torsions are usually considerably smaller than the force constants for the other two types of coordinates and we will ignore possible effects arising from these for the present. The isotopically substituted H atoms in the cases of interest are terminal atoms. One can then show that for a bond stretch involving the isotopic atom, $g_{1, ij} - g_{2, ij} = 1/m_H - 1/m_D$, where the m 's refer to atomic masses. Similarly, for a bend involving one isotopic atom, $g_{1, ij} - g_{2, ij} = (1/m_H - 1/m_D)/R^2$, where R refers to the equilibrium distance between the H atom and the atom to which it is bonded. It follows then from equation (1) that, if the isotopically substituted atoms are involved in the same coordinates in the reactant and in the transition state, there will be no isotope effect, except for $\nu_{1L}^\ddagger/\nu_{2L}^\ddagger$, unless there is a change in a force constant for a coordinate involving an isotopic atom. Moreover, the isotope effect is proportional to such force constant change. Except for any effect on $\nu_{1L}^\ddagger/\nu_{2L}^\ddagger$, geometry changes not affecting isotopic coordinates will not change the isotope effect within the framework of equation (1). Even geometry changes affecting isotopic coordinates will lead to no effect if the bending force constant divided by R^2 is kept constant. We, of course, realize that predictions of equation (1) have to be taken with some caution, but for the present we conclude that one can learn about force constant changes at the isotopic position from a study of secondary isotope effects. We will investigate this conclusion further in the following sections.

ISOTOPE EFFECTS IN THE ABSENCE OF FORCE CONSTANT CHANGES

It is recognized that the vibrational motions of the bonds, bond angles, *etc.* in a molecule are coupled. The question thus arises whether secondary

isotope effects could be caused just from some perturbation of this coupling even in the absence of force constant changes involving the isotopic atom(s). In order to investigate this possibility, we have calculated secondary isotope effects in a number of model systems under the condition that no changes in force constants or geometry take place at the isotopically substituted position⁷. Aside from $\nu_{1L}^\ddagger/\nu_{2L}^\ddagger$, at room temperature and above, such effects are generally calculated to be less than ~ 0.5 per cent per α deuterium, less than ~ 0.2 per cent per β deuterium, and even less for γ and δ deuterium[†]. α here refers to a hydrogen attached to the carbon at which the reaction takes place, *etc.* In *Table 1* results are shown for a model reaction $C_2H_5X + X \rightarrow (C_2H_5X_2)^\ddagger$.

Table 1. Secondary isotope effect in the absence of force constant changes at isotopic positions



T(°K)	MMI	EXC	ZPE	k_H/k_D
50	1.1311	0.9842	1.0425	1.1605
100	1.1311	0.9460	1.0210	1.0925
200	1.1311	0.9204	1.0105	1.0520
250	1.1311	0.9172	1.0084	1.0462
300	1.1311	0.9159	1.0070	1.0432
400	1.1311	0.9151	1.0052	1.0405
500	1.1311	0.9152	1.0042	1.0395
1000	1.1311	0.9164	1.0021	1.0387

^a The corresponding hydrogen reactant is, of course, CH_3CH_2X . A valence force field was used (*i.e.*, no interaction force constants) with $f_{CH} = 4.8$, $f_{CC} = 4.4$, $f_{CX} = 2.9$ millidyne/Å and $f_{HCH} = 0.53$, $f_{HCC} = 0.68$, $f_{HCX} = f_{CCX} = 0.62$, $f_{torsion} = 0.1$ md-Å (one torsional coordinate per CC bond).

^b The transition state force field is the same as the reactant force field with the following exceptions: $f_{HCX}^\ddagger = f_{CCX}^\ddagger = 0.31$ md-Å, $f_{CX}^\ddagger = 2.0$ md/Å and an interaction force constant between the two CX stretching coordinates $f_{12}^\ddagger = 2.0$ md/Å.

At the very lowest temperatures, there is an isotope effect aside from $\nu_{1L}^\ddagger/\nu_{2L}^\ddagger$ for the β -deuterium substitution. However, in the room temperature region, the high temperature limit $\nu_{1L}^\ddagger/\nu_{2L}^\ddagger$ is already closely approached. The isotope effect k_H/k_D is describable as the triple product (MMI) \times (EXC) \times (ZPE) as discussed in the previous paper⁴§. It is interesting to note that in a case where the MMI factor is different from unity, the EXC factor tends to be quite large in the opposite direction even at room temperature. The trend to $\nu_{1L}^\ddagger/\nu_{2L}^\ddagger$ at room temperature is also shown in *Table 2* for α - and γ -hydrogen isotope effects in $n-C_4H_9X$ undergoing an S_N1 type of reaction, in which case we used the force fields with interaction force constants as described in the table. Several calculations have been carried out using very much larger force constants than one would usually use, to show that the results obtained in the "no force constant change" calculations do not depend in some way on a choice of small force constant(s). The results of

[†] One exception of much larger magnitude has been found. An effect deviating from $\nu_{1L}^\ddagger/\nu_{2L}^\ddagger$ by ~ 4 per cent for one α -deuterium substitution has been found under these conditions in the hypothetical reactions: $HCH \rightarrow HC + H$, $DCH \rightarrow DC + H$. This deviation occurred in the case where the CH bond which is broken is enlarged by 33 per cent in the transition state. The effect is understood⁷; it results from the fact that the larger bond distance involves a very light atom which is coupled to the secondary hydrogen atom in a bending motion.

§ Briefly, MMI = mass-moment of inertia factor, EXC = vibrational excitation factor, ZPE = zero-point energy factor.

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or.e such calculation are included in *Table 2*. Finally, the table shows that a secondary carbon isotope effect also approaches the classical limit under the present conditions. For the case of an equilibrium, instead of a reaction, one expects to find and does actually find, in the absence of force constant changes at the isotopic position, that the classical limit $K_H/K_D = 1.000$ is

Table 2. Secondary isotope effects ($k_{\text{light}}/k_{\text{heavy}}$) in the absence of force constant changes at isotopic positions

$T(^{\circ}\text{K})$	A^a	B^b	C^c	D^d
50	1.2518	1.0096	0.8638	1.0397
100	1.1146	1.0067	0.9536	1.0211
200	1.0497	1.0061	0.9931	1.0154
250	1.0395	1.0060	0.9990	1.0148
300	1.0343	1.0060	1.0023	1.0145
400	1.0300	1.0060	1.0058	1.0144
500	1.0285	1.0060	1.0075	1.0143
1000	1.0276	1.0060	1.0106	1.0143
$\nu_{1L}^{\ddagger}/\nu_{2L}^{\ddagger}$	1.0276	1.0060	1.0120	1.0143

^a $n\text{-CH}_3\text{CH}_2\text{CH}_2\text{CD}_2\text{X} \rightarrow (n\text{-CH}_3\text{CH}_2\text{CH}_2\text{CD}_2\text{-X})^{\ddagger}$. A force field with reasonable values for diagonal and also off-diagonal force constants was employed. The transition state differs from the reactant only in that $f_{CX}^{\ddagger} = 0.0$. There are no off-diagonal force constants involving X in either reactant or transition state.

^b $n\text{-CH}_3\text{CD}_2\text{CH}_2\text{CH}_2\text{X} \rightarrow (n\text{-CH}_3\text{CD}_2\text{CH}_2\text{CH}_2\text{-X})^{\ddagger}$. The same force fields were used as in A.

^c $n\text{-CH}_3\text{CH}_2\text{CD}_2\text{CH}_2\text{X} + \text{X} \rightarrow (n\text{-CH}_3\text{CH}_2\text{CD}_2\text{CH}_2\text{X}_2)^{\ddagger}$. All stretching force constants were set equal to 10.0 millidyne/Å, all bending and torsional force constants equal to 5.0 md-Å, and all usually important interaction force constants within the alkyl group equal to 1.0 units. The only non-zero interaction constant involving X occurs in the transition state where the interaction force constant between the two CX stretches is set equal to 10.0 md/Å.

^d $n\text{-CH}_3^{13}\text{CH}_2\text{CH}_2\text{X} \rightarrow (n\text{-CH}_3^{13}\text{CH}_2\text{CH}_2\text{-X})^{\ddagger}$. Only diagonal force constants (of reasonable values) were employed.

approached in the room temperature region. The over-all conclusion of these considerations is that secondary rate isotope effects different from $\nu_{1L}^{\ddagger}/\nu_{2L}^{\ddagger}$ in the room temperature region do not arise just because of the coupling of vibrations in a molecule but must actually reflect something happening at the isotopic position. One should not underestimate the possible magnitudes of $\nu_{1L}^{\ddagger}/\nu_{2L}^{\ddagger}$. It is conceivable that some small normal ($k_H/k_D > 1$) secondary isotope effects could be rationalized as (principally) $\nu_{1L}^{\ddagger}/\nu_{2L}^{\ddagger}$ effects. Such effects would be (fairly) temperature independent.

SECONDARY ISOTOPE EFFECTS AND GEOMETRY OF TRANSITION STATE†

Some years ago, an investigation⁸ was carried out to determine, from isotope effects, the geometry of the transition state in the reaction between hydrogen molecules and chlorine atoms. It was found that from the experimental data, which existed over some limited temperature region, it was not possible to make a decision between a triangular and a linear transition state.

In *Table 3* results are given for the model reaction $\text{CH}_3\text{X} \rightarrow (\text{CH}_3\text{-X})^{\ddagger}$. In the reactant, tetrahedral bond angles were assumed with $R(\text{CH}) = 1.09 \text{ \AA}$ and $R(\text{CX}) = 1.91 \text{ \AA}$. Four different transition states were employed: (i) tetrahedral with $R(\text{CX}) = 1.91 \text{ \AA}$, (ii) tetrahedral with $R(\text{CX})$

† Some preliminary calculations on this topic had previously been carried out by R. Freund, S. Seltzer, and M. Wolfsberg.

= 2.51 Å, (iii) flat CH₃ with R(CX) = 1.91 Å, and (iv) flat CH₃ with R(CX) = 2.51 Å. The same valence force field was used for all four transition states. The CX stretching force constant was set equal to zero in the transition state and the HCX bending force constant was lowered to 0.37 millidyne-Å (from 0.62 millidyne-Å in the reactant). It is seen that the

Table 3. Isotope effect (k_H/k_D) and geometry of transition state CD₃X → (CD₃-X)[‡]

T(°K)	Case ^a					
	1	2	3	4	3'	4'
100	3.66	3.62	4.31	4.28	3.53	3.50
200	1.89	1.89	2.07	2.07	1.89	1.88
250	1.65	1.65	1.76	1.76	1.65	1.64
300	1.50	1.50	1.58	1.58	1.50	1.50
MMI	[1.000	1.012	1.044	1.052	1.044	1.052]
EXC	[0.974	0.970	0.943	0.940	0.955	0.953]
ZPE	[1.541	1.530	1.604	1.596	1.501	1.493]
350	1.41	1.41	1.46	1.46	1.40	1.40
MMI	[1.000	1.012	1.044	1.052	1.044	1.052]
EXC	[0.970	0.965	0.931	0.928	0.946	0.943]
ZPE	[1.449	1.439	1.499	1.493	1.416	1.410]
400	1.34	1.34	1.38	1.38	1.33	1.33
500	1.25	1.25	1.27	1.27	1.24	1.24
1000	1.13	1.13	1.13	1.13	1.12	1.12

^a The cases refer to the transition state geometries discussed in the text: (1) Tetrahedral with R(C-X) = 1.91 Å, (2) Tetrahedral with R(C-X) = 2.51 Å, (3) Flat CH₃ with R(C-X) = 1.91 Å, (4) Flat CH₃ with R(C-X) = 2.51 Å. Unprimed refers to $f_{HCX}^\ddagger = 0.37$ and primed to $f_{HCX}^\ddagger = 0.405$ millidyne-Å. The reactant force constants are: $f_{CH} = 5.1$, $f_{CX} = 2.9$ md/Å; $f_{HCX} = 0.62$, $f_{HCH} = 0.53$ md-Å. In the transition state force field f_{HCX}^\ddagger differs as already indicated and $f_{CX}^\ddagger = 0.0$. R(CH)(transition state) = 1.09 Å. On flat transition state HCX angles = 90°, HCH = 120°.

isotope effect is the same over a wide temperature range for 1 and 2 and for 3 and 4, respectively. On the other hand, it is seen that, if the HCX bending force constant in the transition state is raised to 0.405 md-Å (cases 3' and 4'), then the flattened transition state gives closely the same result as the previous tetrahedral one over a large temperature range. Thus the calculated model isotope effect reflects a lowering of the HCX bending force constant with the amount of lowering to produce a given effect somewhat dependent on geometry. There would, however, be no way of deducing the geometry from a measured hydrogen isotope effect here unless one would know some method of correlating HCX bending force constant with geometry. Such methods may exist, but they are outside of what one can call direct deduction from a measured isotope effect. In Table 3, the decomposition of the isotope effect into its three components is shown at two temperatures. The individual factors tend to be different for 1, 2, 3', 4', while the over-all effect remains the same. Thus the MMI factor is directly related to geometry but there would seem to be no way of obtaining this factor from experimental measurements in the room temperature region. In the subsequent section on temperature dependence, it is explicitly pointed out that it would probably be impossible to differentiate between the flat and tetrahedral transition states here on the basis of temperature dependence measurements in the room temperature region.

If the HCX bending force constant is left the same in the transition state as in the reactant (*i.e.*, 0.62 md-Å), then one finds for the tetrahedral

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transition state that the rate ratio at 300°K differs from $\nu_{1L}^\ddagger/\nu_{2L}^\ddagger$ by less than a factor 1.01, as expected from the previous section. In the case of a flat transition state the corresponding deviation is 1.02 ($\nu_{1L}^\ddagger/\nu_{2L}^\ddagger$ is the same here as for the tetrahedral transition state). As is seen from *Table 3*, if the HCX force constant in the transition state is lowered to 0.37 md-Å, the isotope effects for the tetrahedral and flat transition states differ by a factor 1.05. Similarly, if in the reactant the HCX bending force constant were as low as 0.37 md-Å (and this is rather low), a flat transition state with the same force constant would give rise to an effect which differs from $\nu_{1L}^\ddagger/\nu_{2L}^\ddagger$ by a factor ~ 1.06 at room temperature. Geometries intermediate between tetrahedral and flat have been found to yield intermediate results. There may thus be small isotope effects which come about purely from such geometry changes. These effects are small compared to the magnitude of experimentally determined α -deuterium isotope effects in bond-breaking reactions. (The value of about 1.50 at 300°K in *Table 3* is of the order of what is actually observed in such α -isotope effects⁹.) Explicit mention is made of this case because statements have been made sometimes that the experimental isotope effect here proves a planar configuration in the transition state around the carbon atom which is isotopically substituted.

In *Table 4*, results are presented for a number of transition state geometries as described for the model reaction $\text{CH}_3\text{X} + \text{X} \rightarrow (\text{CH}_3\text{X}_2)^\ddagger$. The same

Table 4. Isotope effect ($k_{\text{H}}/k_{\text{D}}$) and geometry of transition state
 $\text{CD}_3\text{X} + \text{X} \rightarrow (\text{CD}_3\text{X}_2)^\ddagger$

$T(^{\circ}\text{K})$	<i>A</i>	<i>B</i>	<i>Case</i> ^a <i>C</i>	<i>D</i>	<i>E</i>
100	0.1342	0.1318	0.1373	0.1330	0.1477
200	0.3944	0.3930	0.3961	0.3938	0.4128
300	0.5788	0.5782	0.5796	0.5786	0.5934
400	0.7096	0.7093	0.7099	0.7094	0.7193
500	0.8017	0.8016	0.8019	0.8016	0.8079
1000	0.9918	0.9917	0.9918	0.9917	0.9925

^a Reactant force field, diagonal force constants: $f_{\text{CH}} = 4.5$, $f_{\text{CX}} = 4.0$ millidyne/Å; $f_{\text{HCH}} = 0.53$, $f_{\text{HCX}} = 0.53$ md-Å. The same force constants are used in the transition state (which now, however, contains two X atoms) force field with an interaction force constant of 4.0 md/Å between the two CX stretching motions.

Geometries: reactant—R(CH) = 1.09 Å, R(CX) = 1.91 Å, all angles tetrahedral; transition state geometry *A*—R(CH) = 1.09 Å, R(CX) = 2.25 Å, HCH angles = 120°, HCX angles = 90°. Only changes from transition state geometry *A* are listed for other cases: *B*—R(CX) = 1.91 Å; *C*—R(CX) = 3.00 Å; *D*—R(CX₁) = 1.91 Å, R(CX₂) = 2.25 Å; *E*—HCX₁ and HCH angles tetrahedral, XCX angle 180°.

transition state force field was employed in all of the calculations. To the extent that slightly different isotope effects are obtained for different geometries, it seems clear that a small change in transition state force field for one or the other geometry could bring the two geometries into agreement over a temperature range. We mention briefly that we have carried out calculations in which R(CH) was changed in the transition state. As already anticipated in the previous section, there was almost no change in the isotope effect when $f/\text{R}(\text{CH})^2$ was kept constant for all bending force constants involving hydrogens.

It should be noted that the isotope effect in *Table 3* is normal ($k_{\text{H}}/k_{\text{D}} > 1$). This normal effect arises because the HCX bending force constant decreases in the transition state. The isotope effect in *Table 4* is inverse because of a

net increase in HCX bending force constant due to the presence of two X groups in the transition state. Such inverse isotope effects have actually been observed in some reactions of this type¹⁰ although our effects are much larger (in the inverse direction) than the experimental ones. The model isotope effects could be brought into the experimental range by lowering the HCX bending force constants in the transition state. Indeed, they could be lowered so much that positive effects would be observed in the model system. We again emphasize, however, that for the present we are only carrying out model calculations to study the general nature of isotope effects rather than calculations to obtain agreement with experimental results.

Force constant changes between reactant and transition state can, as will be shown subsequently, explain secondary isotope effects over large ranges of magnitude. We have generally found that in the room temperature region, different geometries of transition state will at most lead to slightly different assignments of transition state force constants involving the isotopic atom to explain the experimental effect but that the actual geometry cannot be directly deduced from the effect. We, of course, would not quarrel with the statement that it may be possible then to deduce transition state geometry from these force constants.

TEMPERATURE DEPENDENCE

It is usually possible to express the isotopic rate ratio quite well over a limited temperature range in the form $A \exp(B/T)$. This can even be accomplished in the very high temperature region where we know⁴ that the temperature dependence is actually of the form $C(1 + K/T^2)$. In terms of the decomposition of the rate ratio into its three factors⁴, a rate ratio expression $A \exp(B/T)$ means that, over the limited temperature range, a plot of the logarithm of EXC *versus* $1/T$ yields something resembling a straight line.

In *Table 5*, **A** factors are presented for a number of different model cases. The **A** factors were determined from the points at 300 and 325°K (although in general the point at 400°K is fitted quite well too). In addition to **A** there are listed, at 300°K, the isotopic rate ratio, $v_{1L}^\ddagger/v_{2L}^\ddagger$, and the three factors which make up this rate ratio. Since $k_1/k_2 = (\text{MMI}) \times (\text{EXC}) \times (\text{ZPE})$ and since ZPE is already an exponential, **A** is related to, but not equal to, $(\text{MMI}) \times (\text{EXC})$. The word related must be used because of the temperature dependence of EXC. Thus, in example 11, $(\text{MMI}) \times (\text{EXC})$ is larger than unity while **A** is less than unity. It appears from the table that there is no very direct relationship between individual factors and the **A** factor. Reactions 7 and 8 involve the same force constant changes at the isotopic position in the transition state. The ratio of rate ratios is closely approximated by the ratio of $v_{1L}^\ddagger/v_{2L}^\ddagger$'s and so is the ratio of **A** factors. This effect obviously bears relationship to the cut-off or localization procedure mentioned in the previous paper⁴. Attention is thus called to the fact, which we have not as yet investigated in detail, that the absolute magnitudes of **A** probably bear some relationship to transition state type—*i.e.*, to the value of $v_{1L}^\ddagger/v_{2L}^\ddagger$. Reference should also be made to the previous paper for a discussion of EXC factors.

Table 5. Pre-exponential factors **A** (derived from points 300–325°K) and other relevant information for a variety of model reactions

No.	Isotopic reactant	Reaction type ^a	Force constant changed at isotopic atom(s)	Magnitude of change ^b	A	k_1/k_2 300°K	$\nu_{1L}^{\ddagger}/\nu_{2L}^{\ddagger}$	MMI	EXC	ZPE
1	CD ₂ X	A	HCC	0.62 → 0.37	0.94	1.502	1.079	1.000	0.974	1.541
2	CD ₃ X	A	HCC	0.62 → 0.42	0.97	1.403	1.079	1.000	0.987	1.422
3	CD ₂ X	A	HCC	0.62 → 0.405	0.93	1.496	1.079	1.052	0.953	1.493
4	CD ₂ X	A	CH	5.1 → 4.7	1.06	1.376	1.079	1.000	1.020	1.349
5	CD ₂ CH ₂ X	E	HCC	0.67 → 0.41	0.95	1.503	1.078	1.000	0.990	1.518
6	CH ₃ CD ₂ CH ₂ X	E	HCC _α	0.68 → 0.43	0.96	1.262	1.026	1.001	0.981	1.286
7	CD ₂ X	G	See below ^c	1.17	1.17	0.855	1.086	1.271	0.921	0.730
8	CD ₂ X	I	See below ^c	1.09	1.09	0.791	1.007	1.271	0.889	0.700
9	CD ₃ CH ₂ X	G	CH	4.8 → 4.5	1.03	1.251	1.039	1.131	0.916	1.208
10	¹⁴ CH ₃ CH ₂ X	G	See below ^d	1.03	1.03	1.029	1.027	1.042	0.986	1.002
11	CD ₂ CH ₂ X	G	HCH	0.53 → 0.40	0.98	1.369	1.039	1.131	0.905	1.337
12	CH ₃ CD ₂ CH ₂ X	A	CH	4.5 → 4.2	1.01	1.152	1.016	1.000	1.006	1.145
13	CH ₃ CD ₂ CH ₂ X	A	HCH	0.53 → 0.35	0.99	1.153	1.016	1.000	1.001	1.152
14	CH ₃ CD ₂ CH ₂ X	A	HCC ^e	0.68 → 0.60	0.98	1.150	1.016	1.000	0.995	1.156

^a Details of force fields and geometries will not be given. The force constant changes between reactant and transition state at the isotopic position(s) are indicated. *A* refers to an RX₂[‡] transition state in which the CX bond is ruptured, $f_{CX}^{\ddagger} = 0.0$. *E* refers to an RX₂[‡] transition state in which there is rupture of the CC bond adjacent to the CX bond, $f_{CC}^{\ddagger} = 0.0$. *G* refers to an RX₂[‡] transition state with both f_{CX}^{\ddagger} 's equal and with an equal interaction force constant. *I* refers to an RX₂[‡] transition state with one $f_{CX}^{\ddagger} = 0.0$ and the other f_{CX}^{\ddagger} finite.

^b Stretching force constants in millidyne/Å, bending force constants in md-Å.

^c HCC bending force constants in reactant = 0.62; all HCC bending force constants in transition state = 0.41 md-Å. Attention should be called to the fact that different force constants were employed here than in the model reactions of Table 4.

^d Force constants changed at isotopic carbon: CCX bending force constant, 0.62 → 0.31 md-Å; C-H stretching force constant, 4.8 → 4.5 md/Å (cases 9 and 10 refer to the same force fields).

^e Total of four HCC force constants changed.

The **A** factor for reaction 10 is very close to $\nu_{1L}^\ddagger/\nu_{2L}^\ddagger$. This reaction is a secondary carbon isotope effect and k_1/k_2 is very close to the classical "high temperature" factor $\nu_{1L}^\ddagger/\nu_{2L}^\ddagger$ even at room temperature. The large (> 1) **A** factors in 7 and 8, α -isotope effects in S_N2 reaction types, are interesting but will not be further discussed here. In 1, 2, and 3, **A** factors less than unity are found. These result from a lowering of HCX bending force constant in the transition state. The exact magnitude of **A** does, however, depend on the amount of lowering. On the other hand, in reaction 4, the CH stretching force constant is lowered and an **A** factor larger than unity is obtained. In this case the **A** factor is found to be the same over a wide range of force constant variation. One is led to suspect strongly that careful temperature measurements of secondary isotope effects will enable one to put some limits on the types of force constant changes which occurred. This suspicion finds some confirmation in reactions 12, 13, and 14. Reactions 1 and 3 correspond to the tetrahedral and flat transition state in an S_N1 reaction type previously described (cases 1 and 4' of *Table 3*, respectively). The **A** factors are quite similar and it would appear quite difficult to make a judgement about geometry from the temperature dependence. Reaction 5 shows that **A** (for given k_H/k_D) for a bond-break reaction resulting in CH_3 groups is fairly independent of whether the methyl group is attached to X or to CH_2X . The type of transition state which is employed here (which yields $\nu_{1L}^\ddagger/\nu_{2L}^\ddagger \sim 1.08$ corresponding to fragment masses CH_3 and a very heavy group) is probably a fairly reasonable one for reactions which yield a CH_3 group as a result of a bond rupture. Some encouragement for the opinion that α -deuterium isotope effects in such reactions are largely due to HCC bending force constant changes is furnished by the finding by Zavitsas and Seltzer¹¹ of an **A** factor 0.93 ± 0.02 in the case of a methyl radical formation reaction ($k_H/k_D \sim 1.5$ in the room temperature region).

A very intriguing temperature dependence has been reported in the literature for the β -deuterium isotope effect in the solvolysis of various isopropyl compounds¹². Over the thirty degree temperature range in which these were measured, they were found to be temperature independent. An explanation for this effect has been proposed¹² in terms of different barrier heights for torsional motion of the methyl groups in the hydrogen and deuterium compounds. While such a proposal violates the basic principal of isotope effects that potential functions must be independent of isotopic substitution, it may be possible to recast the argument in terms of anharmonicities. No attempt has been made here to pursue the reasonableness of an argument based on anharmonicities. General considerations of the rate ratio equation lead to the conclusion that it is possible to obtain large temperature-independent isotope effects for a model reaction if some force constants which give rise to small frequencies (*e.g.*, those corresponding to torsions) in the reactant become larger in the transition state while some "large" force constants which give rise to large frequencies (*e.g.*, those corresponding to stretches) in the reactant become smaller in the transition state. One set of model calculations which shows an extremely small dependence on temperature is shown in *Table 6*. It has been found that a fairly wide range of force constants can give rise to such temperature dependences. It is not our desire to intimate that the model force field

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change of *Table 6* is the one responsible for the experimentally observed results. For the time being, it suffices that such temperature dependences should not be regarded as a violation of the usual theory.

Table 6. A "temperature independent" secondary hydrogen isotope effect
 $(\text{CD}_3)_2\text{CHX} \rightarrow [(\text{CD}_3)_2\text{CH-X}]^\ddagger$: (MMI) = 1.0000

$T(^{\circ}\text{K})$	EXC	ZPE	$k_{\text{H}}/k_{\text{D}}$
100	1.0870	1.3503	1.4678
250	1.2493	1.1277	1.4088
280	1.2675	1.1132	1.4110
300	1.2784	1.1053	1.4130
320	1.2883	1.0984	1.4151
340	1.2972	1.0924	1.4170
360	1.3052	1.0870	1.4188
380	1.3121	1.0822	1.4200

^a The geometry of the transition state was taken the same as the reactant (MMI = 1.0). The following force constant changes were assumed: f_{CH} in CH_3 , 4.8 → 3.5 millidyne/Å; f_{torsion} , 0.15 → 1.0 md-Å; and f_{HCC} in CH_3 , 0.68 → 1.0 md-Å. One HCCC torsion coordinate was employed per methyl group.

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We have already demonstrated that secondary α -deuterium isotope effects may involve HCX bending force constants. There may, of course, also be changes in other force constants corresponding to coordinates which involve motion of the isotopic atoms. For β , γ , etc. hydrogen isotope effects, only the off-diagonal force constants and possibly the torsional force constants couple the motion of the isotopic atom directly to the motion of the X group. We shall ignore the possible isotope effects which could result from the changes in the latter force constants and consider changes in CH stretching force constants and HCC and HCH bending force constants involving the isotopic position as possible causes of isotope effects.

$\text{CH}_3\text{CD}_2\text{CH}_2\text{X} \rightarrow (\text{CH}_3\text{CD}_2\text{CH}_2\text{-X})^\ddagger$ was used as the model system. *Figure 1* shows the result on $k_{\text{H}}/k_{\text{D}}$ at 300°K of varying the three above-mentioned force constants independently. The isotope effect has been plotted as a function of $\sum (f_{ij} - f_{ij}^\ddagger)(g_{1,ij} - g_{2,ij})$. It is then seen, with respect to equation (1), that $\bar{\gamma}$ decreases in the order HCC, HCH, CH. This is the result expected⁴ on the basis of a bond vibration approximation since CH vibrational frequencies are larger than HCH frequencies which in turn are larger than HCC frequencies. *Table 7* shows results over a range of temperatures for force constant variations which yield approximately the same values of $k_{\text{H}}/k_{\text{D}}$ at 300°K. Results are shown for a case $k_{\text{H}}/k_{\text{D}} > 1$ (a lowering of the relevant force constant in the transition state). The over-all conclusion from these model calculations is that it may be possible to fit an experimental isotope effect at a given temperature with a variety of different force constant changes. It is to be noted that different types of force constant changes can be combined in different ways. The temperature dependence may help to pin down the force constant change somewhat as

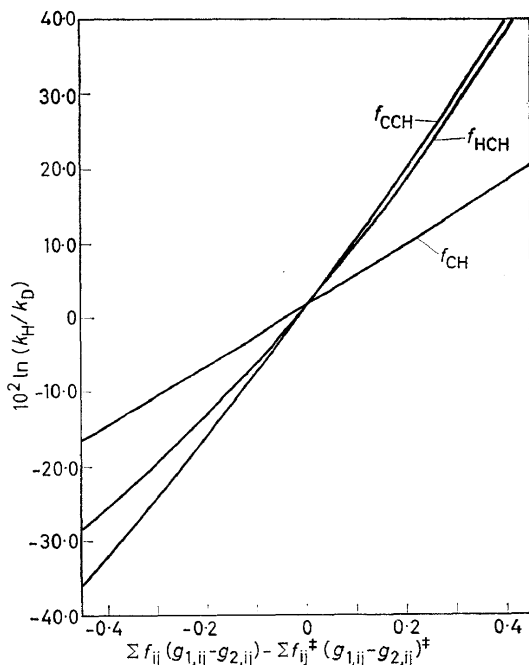


Figure 1. Isotope effect ($k_{\text{H}}/k_{\text{D}}$) as a function of force constant change. $\text{CH}_3\text{CD}_2\text{CH}_2\text{X} \rightarrow (\text{CH}_3\text{CD}_2\text{CH}_2-\text{X})^\ddagger$. The respective labels indicate the respective types of force constants which were changed at the isotopic positions. The units of the abscissa are millidyne \AA^{-1} (atomic mass units) $^{-1}$

noted in the previous section. Thus the **A** factors for the calculations in Table 7 are respectively (as already noted in Table 5, reactions 12, 13, 14): CH stretch lowering, 1.01; HCH bend lowering, 0.99; CCH bend lowering, 0.98. The trend here is in the expected direction. Attention should be directed to the experimental accuracy which is necessary to distinguish

Table 7. Similar isotope effects with different force constant changes^a
 $\text{CH}_3\text{CD}_2\text{CH}_2\text{X} \rightarrow (\text{CH}_3\text{CD}_2\text{CH}_2-\text{X})^\ddagger$; $\text{CH}_3^{14}\text{CH}_2\text{CH}_2\text{X} \rightarrow (\text{CH}_3^{14}\text{CH}_2\text{CH}_2-\text{X})^\ddagger$

$T(^{\circ}\text{K})$	200	300	400	500
(A) f_{CH} , 4.5 \rightarrow 4.2				
$k_{\text{H}}/k_{\text{D}}$	1.2293	1.1518	1.1154	1.0937
$k_{12\text{C}}/k_{14\text{C}}$	1.0191	1.0170	1.0162	1.0158
(B) f_{HCH} , 0.53 \rightarrow 0.35				
$k_{\text{H}}/k_{\text{D}}$	1.2384	1.1527	1.1093	1.0831
$k_{12\text{C}}/k_{14\text{C}}$	1.0204	1.0178	1.0167	1.0161
(C) f_{CCH} , 0.68 \rightarrow 0.60 ^b				
$k_{\text{H}}/k_{\text{D}}$	1.2414	1.1502	1.1050	1.0788
$k_{12\text{C}}/k_{14\text{C}}$	1.0227	1.0193	1.0177	1.0168

^a The transition state force field is obtained from the reactant force field by setting $f_{\text{CX}}^\ddagger = 0.0$ and by making the indicated changes at the isotopic position(s) (millidyne/ \AA for stretch and millidyne- \AA for bend). For the deuterium effect, $\nu_{1\text{L}}^\ddagger/\nu_{2\text{L}}^\ddagger = 1.0155$; for the ^{14}C effect, $\nu_{1\text{L}}^\ddagger/\nu_{2\text{L}}^\ddagger = 1.0143$.

^b Total of four CCH force constants changed.

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such small variations in A . *Table 7* also shows model results for β - ^{14}C secondary effects. The ^{14}C effects are all fairly close to the high temperature limit, $\nu_{1\text{L}}^\ddagger/\nu_{2\text{L}}^\ddagger = 1.0143$, even at room temperature. While there are small deviations of the secondary carbon isotope effects from $\nu_{1\text{L}}^\ddagger/\nu_{2\text{L}}^\ddagger$ which depend both on type and magnitude of force constant change at the isotopic position, these deviations can probably not be regarded as generally useful tools for determining types of force constant changes. It appears to us that, under the proper circumstances, measurement of secondary carbon isotope effects may yield important information about $\nu_{1\text{L}}^\ddagger/\nu_{2\text{L}}^\ddagger$ and thus about the motion of decomposition. Even for a γ - ^{14}C substitution in model reactions of $n\text{-C}_3\text{H}_7\text{X}$, we have found $\nu_{1\text{L}}^\ddagger/\nu_{2\text{L}}^\ddagger$ as high as 1.014 while values as small as 1.000 may also occur.

CONCLUSIONS

It has been shown that, in the absence of any differences between reactant and transition state in the force field or in the geometry directly involving the isotopic position, the secondary isotope effect is well approximated by the classical limit $\nu_{1\text{L}}^\ddagger/\nu_{2\text{L}}^\ddagger$ even at room temperature. When there are force constant changes in coordinates involving the isotopic position, there will be an isotope effect, the magnitude of which depends on the magnitude of the changes. A general decrease of these force constants in going to the transition state will lead to normal isotope effects [$k(\text{light})/k(\text{heavy}) > 1$] while an increase in force constants will generally lead to inverse effects. Different transition state geometries will generally only require slightly different transition state force fields (involving the isotopically substituted position) in order to yield the same isotope effects in the room temperature region[†]. It is proposed that secondary isotope effects serve largely as probes for force constant differences at the isotopic position between reactant and transition state. We again emphasize that other knowledge may, of course, relate force constants to geometry. The temperature dependence of secondary isotope effects need not yield pre-exponential Arrhenius factors of just 1.000. The actual value of this factor may indeed give some clue about force field changes. In general, however, if one only possesses experimental data at one temperature, it will be very difficult to bracket the cause of the isotope effect. Secondary carbon isotope effects may yield very important information about the decomposition motion.

[†] It is conceivable that there may be cases where this geometry-induced small uncertainty in transition state force constants would be bothersome. This question should be discussed when one deals with specific experimental cases.

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